Statistical Component Variability Effects on Oscillator Phase Noise Performance

Patrick A. Green
Northrop Grumman Electronic Systems
Baltimore, Maryland

Abstract—**Component and circuit variations can cause oscillator phase noise degradation. Leveraging component statistical performance such as sustaining stage amplifier residual phase noise, gain variation, resonator Q's, circuit insertion loss, and other component tolerances, a simple model can be produced with these input variables and provide an accurate representation of phase noise performance yields for a given specification or conversely provide a high production yield performance specification. Manufacturers of these components have measured data and/or statistical information regarding many of these parameters. If only measured data exists, there are software tools available that will create probability distributions of the measured data.**

This paper will describe a method on how statistical analysis can be applied to a simple oscillator structure.

I. Introduction

Models for providing phase noise performance are typically inherent to oscillator designers. Not typical in these phase noise models are the statistical component performance parameters that impact oscillator output signal phase noise characteristics. Figure 1 illustrates a simple, but effective mechanism for creating an oscillatory circuit.

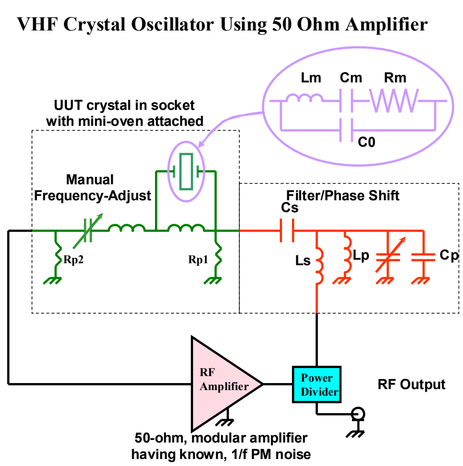

Figure 1 – Simple Oscillator Block Diagram

A simple phase noise model for Figure 1 can be defined mathematically as,

(1) $\mathcal{L}\{f\}=10LOG((k_2/f^2+k_1/f+k_0)((f_o/2Q_Lf)^2+1)+(k_3/f^3))$

Or, in terms of dB

(2) $\mathcal{L}\{f\}=10LOG((10^{(k2dbc@1Hz/10)}/f^2+10^{(k1dbc@1Hz/10)}/f+10^{(floordbc/10)})((f_o/2Q_Lf)^2+1)+(10^{(k3dbc@1Hz/10)}/f^3))$

Where the open loop parameters are as follows;

k_0 - defines the residual white PM noise or floor noise of the sustaining amplifier and circuit losses

k_1 - defines the flicker of PM residual noise of the oscillator sustaining amplifier at 1 Hz

k_2 - defines random walk of the sustaining amplifier at 1 Hz

k_3 - defines the crystal resonator flicker of resonant frequency or self-noise

f_o - Oscillator Frequency

f - Offset Frequency from f_o

Q_L - Crystal Loaded Q

The advent of many software packages lends to the ease of evaluating the statistical variation from sample measurements of components to closed circuit analysis. Enough samples from different lots can be garnered and a probability distribution reflecting their respective performance created from these prolific packages.

II. Initial Circuit Requirements

To initialize an oscillation two conditions must be satisfied. Firstly, the open loop gain of the sustaining stage must exceed the losses within the circuit. Secondly, the phase of the closed loop must have a relationship of $2n\pi$ radians. Under closed loop conditions, the oscillator loop reduces to a unity gain, steady-state value. This usually means that the

1-4244-0647-1/07/$20.00 ©2007 IEEE

sustaining stage will or other nonlinear component will saturate. If so, the open loop gain of the sustaining stage amplifier must exactly match the circuit losses under closed loop conditions.

III. COMPONENT ANALYSIS

A 10MHz Oscillator circuit was used to demonstrate the statistical modeling. A sample of 24 crystals was used to extract their statistical properties. The measured information was a result of evaluation of on-going process improvement of crystal optimization. These results were construed as failures for process optimization; however, the data can be used to provide an example of oscillator performance yields based upon component statistics. The “Q” measured in thousands or “kQ” is shown in Figure 2. Note that the profile is not a normal distribution. This is more representative of an Extreme Distribution skewed to the left. Here, the mean or average is 1.18 million and the Mode (the most likely number) is 1.206 million.

kQ of 10MHz Crystal

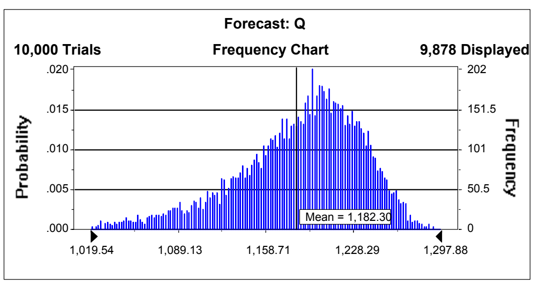

Figure 2 – Probability Distribution of kQ for a 10MHz Crystal

The self noise of the crystals was also measured. Figure 3 reflects the 1Hz phase noise of these crystals. Additionally, since we have a one to one correspondence of the Q and 1Hz phase noise we can additionally obtain a correlation between these two parameters.

10MHz Crystal 1Hz Self Noise

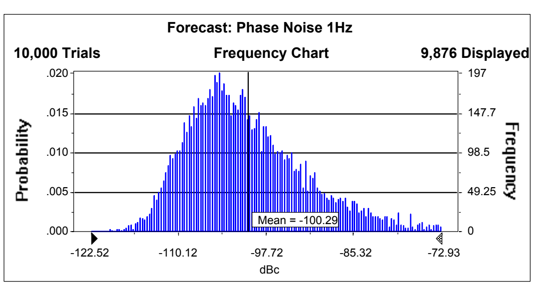

Figure 3 – 1Hz Self Noise of Crystals

The correlation is shown in Figure 4. It is desirable when modeling the output results and defining the probability distributions, it is beneficial to obtain these correlations. These statistical software packages can extract correlation values from measured parameters. Correlation can also be inserted as part of the probability distribution. It can be seen that "kQ" and the 1Hz self noise contains a high correlation of -0.68.

Q vs. 1Hz Strongly Correlated (-0.68)

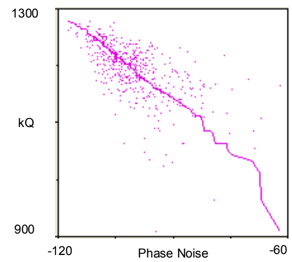

Figure 4 – Correlation factor between Q and 1 Hz Noise

The crystal resistance “Rm” along with the correlation to “kQ” can be obtained. Figure 5 represent the resistance probability distribution function and Figure 6 contains the associated correlation. As expected, an extremely correlated value should exist and does at –0.99.

Resonator Resistance

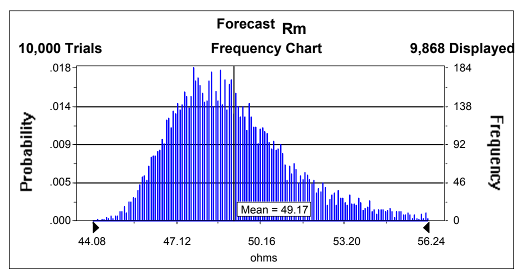

Figure 5 – Probability Distribution for Resonator Resistance

Q vs. R1 Extremely Correlated (-0.99)

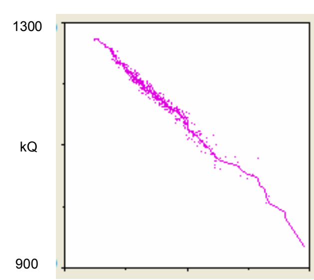

Figure 6 – Correlation factor between Q and Rm

In a series resonant circuit we would terminate each end of the crystal with a resistor. At series resonance, the resonator resistance along with the termination resistors would be representative of a pi-pad. We can now statistically find the insertion loss variation of this circuit during resonance. To simplify we can assume a balanced pad.

The insertion loss using "Rm" profile above and fixed shunt resistances with 10% uniform tolerances results in an insertion loss displayed in Figure 7.

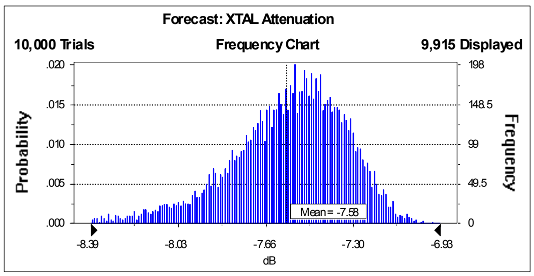

Figure 7 – Crystal insertion loss based upon RM and 10% shunt Resistance

The insertion loss of a two way power splitter from vendors can be garnered from the vendor data sheet. An example for a 10MHz power splitter could be from Mini-Circuits SCPQ-10.5+. It contains a typical insertion loss of 0.15dB over the 3dB power split and can be up to 0.7dB. Measured data can be accumulated to truly define the correct distribution. Since none was available, information from the datasheet can be used to generate either a Normal or a Uniform distribution. Let us assume a Normal distribution centered at 3.15dB with a 1 sigma of 0.18. Since 3σ is approximately 99.7% or about 100%, we can define 1 sigma being 1/3 from max or min from typical. This is (3.7 – 3.15)/3 which yields approximately 0.18. Likewise, from a datasheet the insertion loss of an attenuator for a 5dB attenuator pad is 5±0.5dB. Here, let us use a Uniform distribution.

Link to Mini-Circuits Power Splitter
http://www.minicircuits.com/products/psc_sm_2_90.html

Link to EMC Technology 3dB pad datasheet
http://www.emct.com/microwavepassivecomponents-a42717.html

For convenience, let us assume that these are the total losses construct circuit losses. There are other losses that should be accounted for would be Filter/Phase Shift losses, line losses, mismatch losses. For convenience let us lump these losses in a loss budget using a normal distribution mean of -.7dB with 1σ of 0.1dB.

Selection of the sustaining amplifier is a key to phase noise. Measurements should be taken to elicit performance probability density functions as a function of phase noise under the closed loop operation. In other words, phase noise of the amplifier should be made at the input power and matching conditions it would be scrutinized while in the circuit. An example is shown in Figure 8. Likewise, unit to unit gain variation under these conditions will contribute to the capacity to sustain an oscillation. Figure 9 shows the open loop gain of 18.35dB. It would be operating about 2dB into compression. We shall also take this into consideration and assume a Uniform distribution probability density function.

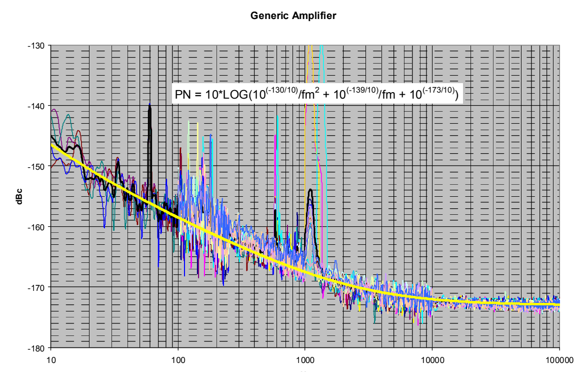

Figure 8 – Generic Example Sustaining Stage Amplifier under closed loop conditions

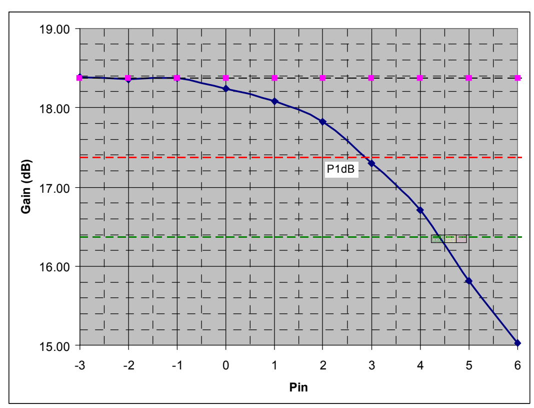

Figure 9 – Open loop sustaining Stage Amplifier gain operating uses a Logistic Distribution of 18.35dB and a scale of 0.2. It will be limited by the power output of the device.

Table I

Open Loop gain Lineup analysis

Component	Distribution Type	Nominal or Mode	Variation or Scale	Min	Max	Sample Data	Gain	4.25	Power Limit	NF	Other Info
Amplifier	Logistic	18.35	0.02			18.39	18.39	20.75	20.75	4.69	-172.31
Splitter	Normal	-3.15	0.18			-3.24	-3.24	17.51		3.24	
Pi-Pad	Uniform	-5	0.50	-5.50	-4.50	-5.02	-5.02	12.49		5.02	
Misc.	Normal	-0.7	0.10	-0.80	-0.60	-0.76	-0.76	11.73		0.76	
Resistor	Uniform	120	10%	108.00	132	122.98		11.73			
XTAL (Rm)	Extreme	48.1	1.82		Max	48.93	-7.52	4.22		7.52	50.71
Resistor	Uniform	120	10%	108.00	132	126.87		4.22			
Total							1.85	4.22		5.76	

IV. CIRCUIT ANALYSIS

The open loop characteristics for the circuit must have excess gain to establish an oscillation. The variation of gains and losses must exceed 0dB. A simple loss budget or gain lineup analysis is shown in Table 1 above. As noted in the open loop gain, Distribution Type indicates what distribution is used. Sample Data (green) is a "single sweep" run. XTAL Loss and Total Gain under the Gain column (blue) can be collected at each of these individual runs and compiled thereby providing a composite distribution. The anticipated open loop gain from this run of 10000 samples is shown in Figure 10.

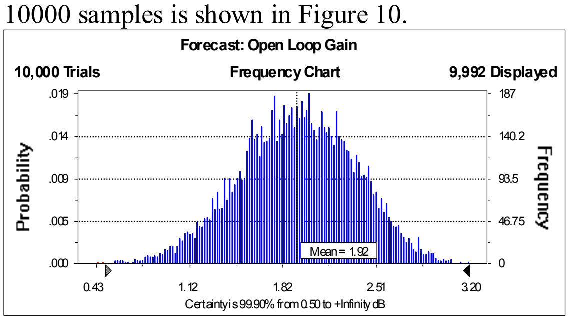

Figure 10 – Predicted Open Loop Gain

Obtaining such an analysis provides some insight for improvements the circuits as well as defining component level performance. Some insights would be circuit yields. Here 99.90% would result in an open loop gain above 0.5dB. If it is desired to have gain greater than 0.5dB, it could easily be garnered. Note that this line up is consists of several items having different probability distribution. Variation over temperatures on each component with its own distribution can readily be provided. It is limited by the circuit modeling of the individual. The second piece of information is the System Noise Figure as a statistical output from Table 1. This would be used to define the floor.

Using equation 2 we as well as other statistical parameters from the open loop gain lineup analysis, amplifier noise performance, crystal self noise, and crystal Q we can write on excel the expected outcome. Table 2 illustrates an example of this particular structure.

Each highlighted green cell in Table 2 represents a statistical distribution. Each section of the equation is represented by the colored arrows. The last column is the expected performance in a closed loop system based on individual component statistical data. The Blue cells represent data to be captured during each sweep. This information is compiled and illustrated in a trend analysis in Figure 11.

Table II

Closed Loop Phase Noise Performance modeled from Equation 2

3 Slope	-300		k3dBc@1Hz=	-92.97
2 Slope	-130.37	=k2dBc@1Hz		
1 Slope	-140.10	=k1dBc@1Hz		
Floor	-171.26	=floordBc		
			QL=	1105.98

Offset Frequency	in dB			in dB		Closed Loop
1	-129.93	1.02E-13	20438499.88	-92.97	5.04E-10	-56.83
3.16	-139.10	1.23E-14	2046798.28	-107.96	1.60E-11	-75.99
10	-147.21	1.90E-15	204385.99	-122.97	5.04E-13	-94.10
31.6	-153.89	4.09E-16	20468.97	-137.96	1.60E-14	-110.77
100	-159.42	1.14E-16	2044.85	-152.97	5.04E-16	-126.30
316	-164.05	3.93E-17	205.68	-167.96	1.60E-17	-140.91
1000	-167.61	1.74E-17	21.44	-182.97	5.04E-19	-154.29
3160	-169.75	1.06E-17	3.05	-197.96	1.60E-20	-164.91
10000	-170.72	8.47E-18	1.20	-212.97	5.04E-22	-169.92
31600	-171.08	7.80E-18	1.02	-227.96	1.60E-23	-170.99
100000	-171.20	7.59E-18	1.00	-242.97	5.04E-25	-171.19
316000	-171.24	7.52E-18	1.00	-257.96	1.60E-26	-171.24
1000000	-171.25	7.50E-18	1.00	-272.96	5.05E-28	-171.25
3160000	-171.25	7.49E-18	1.00	-287.70	1.70E-29	-171.25
10000000	-171.26	7.49E-18	1.00	-298.23	1.50E-30	-171.26
31600000	-171.26	7.49E-18	1.00	-299.93	1.02E-30	-171.26

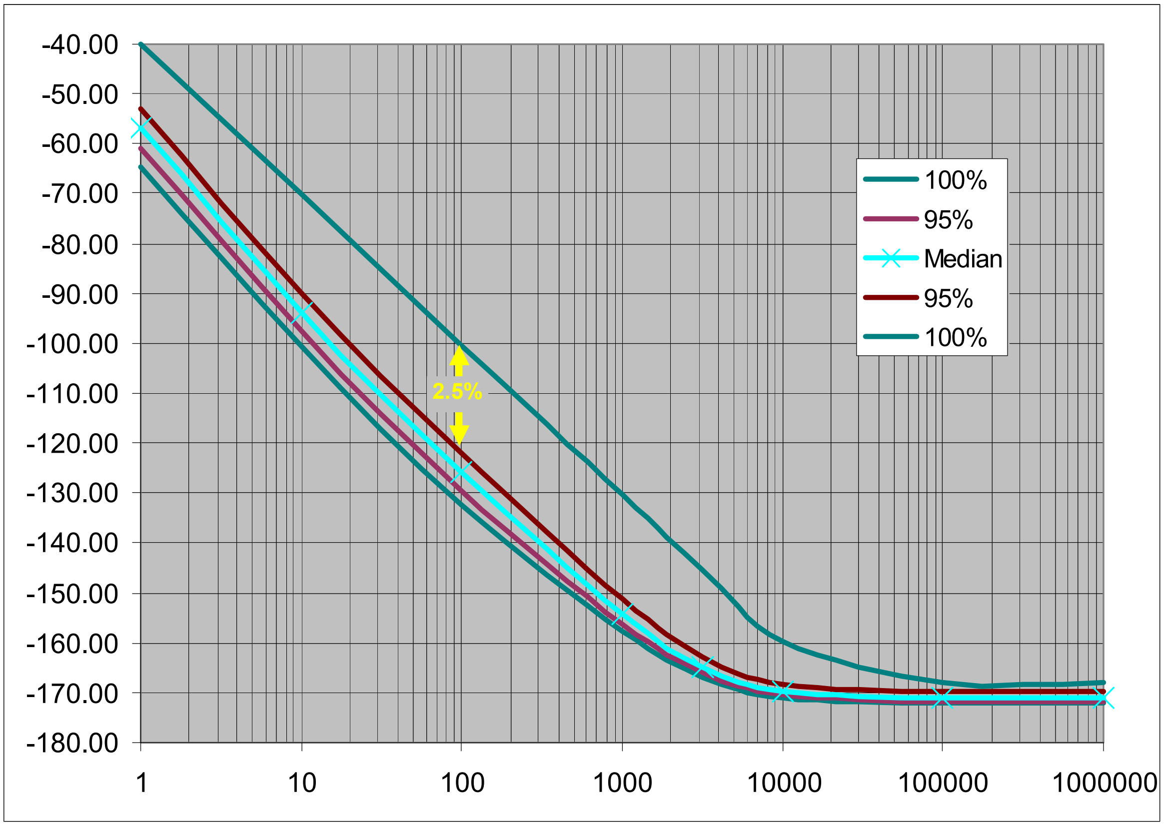

Figure 11 – Trend analysis reflects yield percentages.

V. Conclusions

Many statistical data packages are available. Using these tools extraction of measured components to attain probability distribution functions for each component is simple. Furthermore, evaluation of different aspects of circuit parameters can be evaluated as a statistical yield measurement. Different probability distribution functions beyond typical normal and uniform distributions are readily obtainable. Performance analysis using these statistical tools can be extremely useful in determining yields. It also can provide sensitivity of key components. These component probability distributions also contain correlations between key performance parameters reflecting true component characteristics. Finally, models can be created to provide further enhancements, such as individual component temperature variants on gain/loss, noise figure, third order intercept, etc. Circuit probability output distribution can be created and used as a single line item in a higher level of integration.

Several commercially available statistical software packages/tools that can be used are Crystal Ball by Decisioneering, MegaStat, and MiniTab.